ARITHMETIC MADE EASY

Made Easy Series

ARITHMETIC MADE EASY

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PREFACE

SOME one once made a bad pun about arithmetic.

"Children need to cipher, but it is not a thing they sigh fer."

A sad joke; a sadder truth.

Alas, children — sometimes, too, those of an older growth — do indeed often sigh, not *for*, but *over* the Science of Numbers.

Yet, they need not. As a matter of fact, there is nothing in arithmetic to overtax any average brain. Nor should the study prove tedious. A fair amount of attention, a fair amount of practice, mean easy and rapid mastery.

The great trouble has been heretofore in the way of teaching. The whole subject has been made needlessly hard at the outset. The chief fault has been in too many definitions at the start. The pupil has been forced

to study the meaning of so many strange words as to tire his brain before he has had a chance to grasp the first principles of the science. It is as if he had to learn a new language, and by just so much his difficulties in studying arithmetic have been increased.

In this book the author has tried to correct that evil, and to tell only what is necessary, in the simplest form, unburdened by a terrifying terminology.

Also, hitherto, the student of arithmetic has been faced with an excess of rules that must be committed to memory, parrot-wise. This has tended toward making the pupil depend on the mechanics of memory rather than on exact understanding of the principles involved. When one actually does understand the principles on which the science is founded, the rules will take care of themselves.

In the present volume, the writer has sought to make the principles so plain as to

render needless the memorization of rigid rules.

It is hoped that by such simplicity the way is cleared for the student, and the study made pleasant as well as profitable.

And that word, "profitable," brings home to us the vast importance of arithmetic. The science of numbers was of vital importance to the primitive man, who counted his flocks and herds, and bartered of them for the goods of the trader. But as civilization has grown throughout the world with the passing centuries, arithmetic has developed as the base on which almost everything else has been builded. Without it, there could be no other science, and little art. Without the exact calculations of arithmetic, there could be no chemistry; the great truths of astronomy would be forever hidden from us. To-day, every department of human life depends on our mastery of arithmetical knowledge. By means of it, we are able to build

our mighty bridges, to tunnel under rivers and through mountains. The enormous resources of our country have been opened up by the building of the railways, which throughout depend on figuring — surveys, capacity of cars, locomotive power, train-dispatching, and so on for endless details of construction and operation.

So, too, arithmetic plays its part in both the making of ships and in their navigation. Arithmetic is the magic by which the skipper learns his precise whereabouts though the ship may be a thousand miles from any land.

We add, with infinite regret, that the hideous weapons of present-day warfare could not have their being were it not for a monstrous science based on arithmetical calculations.

A facility at figures is of advantage to any one, — man, woman or child. Without some little knowledge of the science, no one can go about his affairs intelligently. The

greater the skill, the greater the ability. A good understanding of arithmetic is a very valuable asset to any one who means to make a success of life. And the study of it need not be tiresome. The student should go forward slowly enough to be sure of his mastery at each step of the way. He should set out to gain a thorough understanding of each principle. He will find that in such understanding there is an ever-increasing pleasure — a pleasure not to be had in the tedious memorizing of endless rules. That pleasure comes from the sense of steadily growing mental power.

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NUMERATION

ARITHMETIC MADE EASY

Chapter I

NUMERATION

ARITHMETIC is the science of numbers. The naming of numbers is called numeration.

Every child learns the names given to the twelve hours of the day. Having learned these, he has really mastered the hardest part of numeration. For the names of the numbers from one to twelve are arbitrary.

The names of all the numbers from thirteen upward are compounds. These compounds are made from various combinations of ten and the figures from one to nine. Thus ten and three are put together and called thirteen. Ten and four are put together and called fourteen.

In all the numbers ending in *teen*, this syllable simply stands for ten. Fifteen is the name for five and ten; sixteen, for six and ten; eighteen, for eight and ten; nineteen, for nine and ten.

When two tens are reached, the name used is twenty. This is made up of two and ten, the ten in this case being shortened to *ty*.

Twenty is compounded with the figures from one to nine. Thus: twenty-one, twenty-two, and so on.

In the same way, the name given to three tens is thirty. Here, again, compounds are made with the figures from one to nine. Thus : thirty-one, thirty-two, and so on.

The name forty is given to four tens; fifty, to five tens; sixty, to six tens; seventy, to seven tens; eighty, to eight tens; ninety, to nine tens.

But when ten tens are reached, a distinctive name is given to the number. This name is a hundred.

Two hundred means twenty tens. The other hundreds follow in like fashion up to nine.

But the tenth hundred has its own distinctive name. This name is a thousand. The thousands are counted in their turn from one up to nine hundred. But ten hundred thousand is given a new name. This name is a million.

Thereafter, each increase of the number by three figures receives a special name. These are, in their order: billions, trillions, quadrillions, quintillions, etc.

The figures from one to nine are termed units.

Units: 1, 2, 3, 4, 5, 6, 7, 8, 9.

The right-hand figure in any number is said to be in the units column. Thus, in the number 15, the 5 is in the units column.

Ten is indicated by the figure one, followed by a cipher. This cipher is in the units column. The figure one is in the tens column.

And always the figure next to the units figure is in the tens column.

The same system is applied for twenty and its compounds. Here, the two is in the tens column, being written before a cipher in the units column.

The same method is followed with thirty, forty, and so on, up to ninety.

But when one hundred is reached, the figure one is removed one place further to the left by the addition of a second cipher. The one is now said to be in the hundreds column.

The figure one is removed one place further to the left by the addition of a third cipher, when a thousand is reached.

The same system is followed for the higher numbers — millions, billions, etc.

For convenience in reading the higher numbers, each three places, counting from the right to the left, is often marked by a comma. Thus: 3,456,500

The Columns

1	2	3	4
thousands	hundreds	tens	units

Decimals

Our system of figures has ten as its basis, probably because man originally counted on the ten figures of his hands. This principle has already been shown in reference to whole numbers. It applies equally to decimals. But where in whole numbers the progression by tens is from the right to the left, in decimals it is from the left to the right. The point of distinction between the whole number and the decimals is indicated by a period, called the decimal point.

The word "decimal" means a fraction of ten. It is most easily illustrated by our money standard. In this, the dollar is the unit. The dollar is divided into one hundred cents. The decimal point separates the

number of dollars from a following fractional part of a dollar. Thus, one dollar and ten cents is written \$1.10. The one following the decimal point indicates that the number is one tenth of one hundred. For convenience, the cipher is added, so that we say ten cents; that is, ten one-hundredths of a dollar. To write two cents in figures, we place a cipher after the decimal point before the figure two, which thus occupies the hundredths place: \$0.02.

The tenth of a cent, one mill, would be written in the third place after the decimal, the thousandth, and it would be preceded by two ciphers: \$0.001.

By bearing in mind that in every case the value of decimal fractions decreases from the decimal point to the right by tens, there will be no difficulty in reading or writing any such number. The first place following the decimal point is for the tenths; the second, for the hundredths; the third, for the thou-

sandths; the fourth, for the ten-thousandths; and so on.

For example, .0006 stands for the fraction, six ten-thousandths.

NOTE

Instead of figures, letters are sometimes used to express numbers, as, for example, on a clock dial, or in the chapter headings of this book. A list of the Roman numerals, as these are called, is included among the tables at the end of the volume.

ADDITION

Chapter II

ADDITION

YOU have five fingers on one hand. You have five fingers on the other hand. These together make ten fingers. That is addition. Five and five make ten. When two or more numbers are thus put together the result is called the sum.

The word indicating addition is "plus." We say, "Five plus five equals ten." The sign + is used in place of the word "plus"; and the sign = represents the word "equals." So, for five plus five equals ten, we write $5 + 5 = 10$.

The method of addition in its simplest form is as follows:

The various numbers are written one below another in columns. The units of each number form the first column, with the tens

next on the left, and the hundreds, etc., in due order.

The first addition is of the units column. The unit in the sum is set down under the units column, but if the sum be ten or more, the tens are used as a new figure in the tens column, and to it the other figures in the tens column are added.

$$\begin{array}{r} 5 \\ \underline{5} \\ 10 \end{array}$$

The like method is followed in writing down the sum of the tens and mentally carrying any hundreds to the hundreds column, to serve as the first figure in the next addition. The same course is followed with the other columns, to any extent required.

$$\begin{array}{r} 3281 \\ 4069 \\ 110 \\ \underline{7504} \\ 14964 \end{array}$$

Decimals present no difficulty in addition. Each column, beginning with that on the right, is added singly, with the carrying-over process just as in the case of whole numbers.

$$\begin{array}{r} 1.5 \\ \underline{1.5} \\ 3.0 \end{array}$$

$$\begin{array}{r} .741 \\ \underline{.268} \\ 1.009 \end{array}$$

Facility in addition is to be gained only by practice. But a very moderate amount of practice will afford astonishing results. The student should familiarize himself with the simpler forms of addition until they are firmly fixed in memory. He should practice the adding of two to the other higher numbers up to a hundred. He should do the like with the other units, and then with the tens. In

a short time he will know, without stopping to think, that not only do two and two make four, but also that thirteen and eighteen make thirty-one, and with like ease of any simple addition.

SUBTRACTION

Chapter III

SUBTRACTION

SUBTRACTION is the reverse of addition. If we take away five fingers from the total of our fingers, the remainder is five fingers. In this process the subtraction is expressed by the word "minus," which means less. Thus we say, "Ten minus five equals five." The minus is represented by the sign $-$. So we have: $10 - 5 = 5$.

The number from which another number is taken away is called the minuend. The number subtracted from the minuend is termed the subtrahend.

When the minuend is smaller than the subtrahend, the method of subtraction is reversed. The minuend itself is subtracted from the subtrahend, and the difference, or remainder, is preceded by a minus sign.

Here is an illustration as to the minus number. A man desires to purchase a railway ticket costing twenty-five dollars. On counting his money, he finds that he has only twenty-three dollars. This is two dollars short of the required sum. That is to say, the result of his calculation is -2.

Subtraction is, in a way, simpler than addition, from the fact that it concerns itself with only two numbers at a time, while addition may include in one process a long series of numbers.

In setting down the minuend and the subtrahend beneath it, the units, tens, etc., should be placed in their respective columns.

The units of the subtrahend are taken away, or subtracted from those of the minuend, and the remainder is written in the units column of the answer. Thus, with the minuend five and the subtrahend four, the difference, one, is the result of the subtraction.

$$5 - 4 = 1$$

Or:

$$\begin{array}{r} 5 \\ \underline{4} \\ 1 \end{array}$$

When the unit of the subtrahend is larger than that of the minuend, the minuend unit is mentally increased by ten, and the subtraction is then made. Then, in the next step, the tens of the subtrahend are increased by one, when the subtraction is made. The process is the same in connection with the hundreds of other columns.

In subtracting twenty-four from thirty, the unit of the subtrahend is four. This is to be taken away from the cipher of the minuend. We therefore add ten to the minuend, and then subtract four from this ten. To compensate for this, when we perform the subtraction in the tens column, we increase the two of the subtrahend by one. This makes the tens number of the subtrahend three, which leaves no remainder from the three of the minuend.

$$30 - 24 = 6$$

$$\begin{array}{r} 30 \\ \underline{24} \\ 6 \end{array}$$

For convenience in the working out of problems, it should be borne in mind that the same number added to, or subtracted from, both the minuend and subtrahend leaves the remainder unchanged.

$$30 - 20 = 10$$

$$\begin{array}{r} 30 \\ \underline{20} \\ 10 \end{array}$$

Adding ten to both the minuend, thirty, and the subtrahend, twenty, we still have the same remainder.

$$40 - 30 = 10$$

$$\begin{array}{r} 40 \\ \underline{30} \\ 10 \end{array}$$

Or, subtracting ten from both the minuend, thirty, and the subtrahend, twenty:

$$20 - 10 = 10$$

$$\begin{array}{r} 20 \\ \underline{10} \\ 10 \end{array}$$

The accuracy of the result in any subtraction is easily tested. The remainder added to the subtrahend must equal the minuend. Also, subtracting the remainder from the minuend gives the subtrahend.

$$15 - 3 = 12$$

$$\begin{array}{r} 15 \\ \underline{3} \\ 12 \end{array}$$

If now we add the remainder and the subtrahend, we have the minuend as a result.

$$3 + 12 = 15$$

$$\begin{array}{r} 3 \\ \underline{12} \\ 15 \end{array}$$

Likewise, the subtraction of the remainder from the minuend gives the subtrahend.

$$15 - 12 = 3$$

$$\begin{array}{r} 15 \\ \underline{12} \\ 3 \end{array}$$

MULTIPLICATION

Chapter IV

MULTIPLICATION

WHEN we add five to five, making ten, and then add another five, making fifteen, we are multiplying. For in this illustration, the sum, fifteen, is obtained by three fives added together. Three times five makes fifteen. So, also, three times two gives six.

$$5 + 5 + 5 = 15$$

$$2 + 2 + 2 = 6$$

The sign used to indicate multiplication is $\times$, which takes the place of the word "times."

Now, instead of the longer form by addition, we write:

$$3 \times 5 = 15$$

The number repeated is called the multiplicand. The number of repetitions of the

multiplicand is shown by the multiplier. The result of the multiplication is termed the "product."

Sometimes, the multiplicand and the multiplier are called "factors," when they are considered in relation to their product. For example, two and fifteen are factors of thirty, since thirty is the product of two times fifteen. Five and six, also, are factors of thirty, since five times six makes thirty.

It is obvious that the same result will be secured whichever of the two numbers be regarded as the multiplicand and multiplier respectively. Thus, three times five makes fifteen; and five times three, as well, makes fifteen.

$$3 \times 5 = 15$$

$$5 \times 3 = 15$$

Or:

$$3 \times 5 = 5 \times 3$$

When numbers appear as above with the sign =, the whole is called an equation. That is to say, the numbers on one side of

the sign equal those on the other. Any change to one side, without a corresponding change to the other, destroys the equation — the equality.

Here is an equation:

$$5 + 6 \times 3 = 23$$

If it were changed to read:

$$5 + 6 \times 3 - 2 = 23$$

it would be untrue.

In connection with the equation above, it should be noted that the sign of multiplication has no effect on any number except the one next to it. It cannot reach beyond a plus or minus sign.

So, in the illustration given, the five is added to the product of six multiplied by three, and the result is twenty-three. If the five were added to the six, and the multiplication by three followed, the product would be thirty-three, which would be a wrong reading of the equation.

When it is necessary to multiply numbers joined by the plus or the minus sign, a parenthesis is used.

$$5 + 6 \times 3 = 23$$

But:

$$(5 + 6) \times 3 = 33$$

In the second equation the five and the six within the parenthesis are first added together and their sum is then multiplied by three.

The student must memorize all the products possible in the multiplication of the numbers from one to twelve. For convenience in studying these, the following table is given.

A careful study of this table will be of help for practice of addition as well as of multiplication.

The table shows the products of multiplication from

$$1 \times 1 = 1 \text{ to } 12 \times 12 = 144.$$

It should be studied so thoroughly that its facts will be as familiar in the mind as the letters of the alphabet. The numbers

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

in the top row, from one to twelve, serve as the multipliers, while the numbers in the left-hand column are the multiplicands. Place a finger on the seven square in the upper row. Then move the finger downward one

square in the same column. The figure two appears in the left margin of this row. That means, seven times two is fourteen. For fourteen is the number in the square on which your finger now rests. Moving the finger down one square again in the same column, the number is twenty-one. The figure in the left margin is three. Seven times three makes twenty-one.

In the case of any two numbers, one in the top row and one in the left column, the product is shown in the square found at the point of intersection of a line drawn down vertically from the number in the top row and a line drawn horizontally from the figure in the left column.

In multiplying, it is usually more convenient to use the smaller number as the multiplier, placing it directly under the multiplicand, unit under unit, etc. Thus, two times one hundred and twenty-three would be set down as follows:

$$\begin{array}{r} 123 \\ 2 \\ \hline 246 \end{array}$$

Where tens result from multiplication of the units, they are mentally carried over to the tens column, and the other orders are treated in the same manner.

In multiplying one hundred and twenty-three by four, we find the product of the units, four times three, to be twelve. We therefore set down two as the units figure in the product. The next step in the multiplication, four times two, gives eight, to which is added the one mentally carried over to the tens column. Thus, the second figure of the product is nine.

$$\begin{array}{r} 123 \\ 4 \\ \hline 492 \end{array}$$

When the multiplier contains two or more figures, the multiplication involves also addition.

The multiplier is set under the multiplicand with the units, tens, etc., in their respective columns. The first step is the multiplication of the multiplicand by the units figure in the multiplier.

For example:

$$\begin{array}{r}
123 \\
12 \\
\hline
246
\end{array}$$

We first have 246; $123 \times 2 = 246$.

The next step is the multiplication by the tens figure in the multiplier, which in this case is one. So, we have for the second product, 123.

$$123 \times 1 = 123.$$

This is set down under the first product, but removed one place to the left, since it is a multiplication by the tens figure. Then these two products are added together.

$$\begin{array}{cccc}
 & 1 & 2 & 3 \\
 & & 1 & 2 \\
\hline
 & 2 & 4 & 6 \\
1 & 2 & 3 & \\
\hline
1 & 4 & 7 & 6
\end{array}$$

Should there be three figures in the multiplier, there will be three products to be added together, and there will be a corresponding increase in the number of products as the number of figures in the multiplier increases.

There is no difficulty with decimals in multiplication. It is necessary only to bear in mind that the product will contain as many decimal places as there are in the multiplier and multiplicand added together.

DIVISION

Chapter V

DIVISION

DIVISION is the process by which we find how many times one number is contained in another. The number that contains the other is called the dividend, and the number contained in the dividend is called the divisor. The number of times the divisor is contained in the dividend is called the quotient. Thus, sixteen divided by four gives four; for four is contained four times in sixteen. Here sixteen is the dividend, four is the divisor, and four is the quotient.

The sign $\div$ is used to signify divided by. So we have: $16 \div 4 = 4$.

Sometimes it is convenient to write the divisor under the dividend with a line between: $\frac{16}{4}$ instead of $16 \div 4$.

After one has mastered the multiplication table, problems in division are not difficult, for they present only a reversal of the facts in multiplication. How much is sixteen divided by four?

We already know from the multiplication table that four times four equals sixteen. Therefore, the answer to the question of division is four. Always, when the quotient and the divisor are multiplied together the product will be the dividend.

If one is familiar with the multiplication table, when the divisor is fifteen or less, the figure or figures in the quotient can be set down from memory, and no other calculation is necessary.

To illustrate: Let us take the number three hundred and forty-five and divide it by fifteen. We set down the figures thus:

$$15\,\underline{\big|\,345\,}$$
$$23$$

In this example we look to see how many times the divisor is contained in the first two figures of the dividend, thirty-four. It is contained two times, since two times fifteen is thirty. There is a remainder of four. This four in the tens column with the five in the units column gives forty-five. From our knowledge of the multiplication table, we know that fifteen is contained in forty-five three times, since three times fifteen is forty-five. We therefore set down three as the units figure in the quotient.

With numbers beyond fifteen a longer method is necessary. As follows:

```
        16
    ------
731)11696
    731
    ----
    4386
    4386
    ----
```

It will be observed that the divisor is written to the left of the dividend, as in the preceding example. But the quotient is

written above the dividend, in order to leave the space below the dividend clear for the other figures necessary.

The simplest procedure in solving the problem is first to consider the seven of the divisor as being contained once in the first two figures in the dividend, eleven. From this, it is easy to perceive that the seven hundred and thirty-one, also, will be contained once in the first four figures of the dividend. We, therefore, write down one as the first figure of the quotient. In doing this, care should be taken to write it above the nine of the dividend, in the tens column, since the unit in the dividend has not as yet been considered.

The next step is to multiply the divisor by the one already found as a first figure in the quotient. The result is set down under the dividend, writing it so that the one is under the nine in the dividend. This is necessary as suggested above, because the

unit in the dividend has not yet had attention.

$$1169 - 731 = 438$$

To this remainder, the unit in the dividend, six, is now added. Thus we have:

$$4386 \div 731 = 6$$

This six is set down in the quotient after the one already found.

$$11696 \div 731 = 16$$

That this quotient is correct may be verified thus:

$$731 \times 16 = 11696$$

Or:

$$11696 \div 16 = 731$$

When there are decimals in the dividend, but not in the divisor, the decimal point must be put in the quotient as soon as it is reached in the dividend.

If both dividend and divisor contain decimals, those in the dividend may be regarded

as whole numbers, by moving the decimal point in the divisor a corresponding distance to the right.

Where the divisor contains decimals, while the dividend does not, a decimal point should be added to the dividend, followed by as many ciphers as there are decimal places in the divisor. Thus, the decimals neutralize each other.

When the divisor is not contained an even number of times in the dividend, the quotient may be carried on as far as desired in decimal places by adding to the dividend a decimal point, followed by the required number of ciphers.

It should be noted that the multiplication of a divisor and a dividend by the same number has no effect on the quotient. This applies also to the division of the two by the same number.

NOTE

Many prefer to write the quotient to the right of the dividend, separated from it by a vertical line.

FRACTIONS

Chapter VI

FRACTIONS

WHEN a unit is subdivided into equal parts, these parts are called fractions. Thus one-half of any unit is a fraction. It is written: $\frac{1}{2}$. So, also, a quarter of any unit is termed one fourth, and is written: $\frac{1}{4}$.

A unit may be divided into any desired number of parts, and the fraction similarly written.

In any fraction the lower figure (or figures) indicates the number of parts into which the unit is divided. It is called the denominator. The number taken of such parts is shown by the upper figure (or figures), named the numerator. For example, in the fraction $\frac{7}{8}$, the eight shows that the unit is divided into eight equal parts, and is the denominator.

The seven shows that seven of these parts are taken, and it is the numerator.

When the numerator is less than the denominator, the fraction is called a proper fraction. When the numerator equals the denominator, or exceeds it, we have an improper fraction. $\frac{9}{10}$ is a proper fraction; $\frac{11}{10}$ is an improper fraction.

A whole number followed by a fraction is called a mixed number. $3\frac{3}{4}$ means three units and, in addition, three quarters of a fourth unit. Its value is four, less one fourth. So, we might write the equation:

$$3\tfrac{3}{4} = 4 - \tfrac{1}{4}$$

It is obvious that all improper fractions represent a whole number or a mixed number. This equivalent of the improper fraction is found when the numerator is divided by the denominator.

$$\tfrac{15}{3} = 15 \div 3 = 5$$

$$\tfrac{16}{3} = 16 \div 3 = 5\tfrac{1}{3}$$

In the case of $\frac{16}{3}$ the division leaves a remainder of one, so that we add $\frac{1}{3}$ to the 5 of the quotient.

Any mixed number may be turned into an improper fraction by multiplying together the whole number and the denominator of the fraction, then adding the numerator of the fraction to this product, and writing it as the numerator over the denominator.

Thus: To reduce $5\frac{3}{4}$ to an improper fraction, we multiply together the whole number, 5, and the denominator of the fraction, 4. To this product, 20, we add the numerator of the fraction, 3. The sum, 23, is written as the numerator of the improper fraction, over the denominator, 4.

$$5\frac{3}{4} = \frac{5 \times 4 + 3}{4} = \frac{23}{4}$$

A fraction may be simplified without altering its value by dividing both the numerator and the denominator by the largest divisor common to both. For example, $\frac{10}{20}$ may be

reduced to $\frac{1}{2}$, since 10 as a common divisor is contained once in the numerator and twice in the denominator.

So, too, multiplication of both numerator and denominator by the same number makes no change in the value of the fraction, although its terms are altered.

To multiply together a whole number and a fraction, we first find the product of the whole number and the numerator of the fraction, then divide by the denominator of the fraction.

$$7 \times \tfrac{3}{8} = \tfrac{21}{8} = 2\tfrac{5}{8}$$

In multiplying fractions, we find the product of the numerators for a new numerator, and the product of the denominators for a new denominator.

$$\tfrac{1}{2} \times \tfrac{3}{5} = \frac{1 \times 3}{2 \times 5} = \tfrac{3}{10}$$

Before multiplication, any mixed number should be reduced to an improper fraction.

The process of multiplication is often

greatly simplified by the cancellation of factors common to any numerator and denominator. This cancellation should take place before the multiplication. In some instances it will render mutiplication unnecessary. As, for example, in the following instance:

$$\frac{\overset{3}{\cancel{9}}}{\underset{5}{\cancel{10}}} \times \frac{\cancel{2}}{\cancel{3}} = \frac{3}{5}$$

In the above, an examination of the two fractions, $\frac{9}{10}$ and $\frac{2}{3}$, shows that we may divide the numerator of the first, 9, and the denominator of the second, 3, by 3. The 3 is contained in the 9 three times, and we therefore cancel the 9, and write 3 in its place as a new numerator. The denominator, 3, contains the same divisor once. Therefore, 1 may be written as the denominator of the second fraction. As a matter of fact, however, it is not necessary to write down the 1 in place of the canceled 3, because 1 has no value in

the multiplication. It is simpler merely to cancel the 3.

In like fashion, we find that 2 is a common divisor for the denominator of the first fraction and the numerator of the second. Thus, the new denominator of the first fraction becomes 5, the ten being canceled, and the numerator of the second fraction is canceled. We have remaining the fraction $\frac{3}{5}$, which is in fact the product of multiplying together $\frac{9}{10}$ and $\frac{2}{3}$.

This may be tested as follows:

$$\frac{9 \times 2}{10 \times 3} = \frac{18}{30} = \frac{18 \div 6}{30 \div 6} = \frac{3}{5}$$

Since division is actually the reverse of multiplication, in order to divide a fraction by a fraction, we merely reverse the terms of one fraction, and then mutiply the two fractions together.

Let us consider $\frac{1}{2} \div \frac{1}{4}$. That is to say, how many times is a quarter contained in a half?

We know that the answer is two. Two quarters of a dollar equal one half dollar, and so the fourth part of the dollar is contained in the half two times. Now, writing this in fractions, we have:

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$$

Or, by cancellation, the process would be simplified thus:

$$\frac{1}{2} \div \frac{1}{4} = \frac{\cancel{1}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{1}} = 2$$

The method of division is again shown in the following illustration:

$$\frac{7}{10} \div \frac{4}{5} = \frac{7}{10} \times \frac{5}{4} = \frac{35}{40} = \frac{7}{8}$$

Or, by cancellation:

$$\frac{7}{10} \div \frac{4}{5} = \frac{7}{\underset{2}{\cancel{10}}} \times \frac{\cancel{5}}{4} = \frac{7}{8}$$

For the addition and also the subtraction of fractions, it is necessary to secure a common denominator. This can be done by finding the smallest number that will contain an even number of times each of the denomi-

nators, and writing the common multiple as a new denominator for each fraction. A new numerator is secured for each fraction by multiplying the original numerator by the number of times the original denominator is contained in the new denominator.

Take the fractions $\frac{7}{8}$ and $\frac{3}{9}$. The lowest common denominator for these is 72, since it is the smallest number that will contain the denominator, 8, and the denominator, 9, without remainder. So, 72 becomes the new denominator for each fraction.

The original denominator of the first fraction, 8, is contained in 72 9 times. We therefore multiply the numerator, 7, by 9 for the new numerator. Thus we secure $\frac{63}{72}$ in place of the original first fraction. The original denominator of the second fraction, 9, is contained in 72 8 times. So, we multiply the numerator, 3, of this fraction by 8, which gives 24, and we write $\frac{24}{72}$ in place of the original fraction.

Now, if we desire to add $\frac{7}{8}$ and $\frac{3}{9}$, we secure the common denominator, as above indicated, and then add the numerators, and set down their sum over the common denominator. In subtraction one numerator is taken from the other, and the remainder is set down over the common denominator.

$$\frac{7}{8} + \frac{3}{9} = \frac{63}{72} + \frac{24}{72} = \frac{87}{72} = 1\frac{15}{72} = 1\frac{5}{24}$$

$$\frac{7}{8} - \frac{3}{9} = \frac{63}{72} - \frac{24}{72} = \frac{39}{72} = \frac{13}{24}$$

Where there are whole numbers or mixed numbers combined with fractions for addition or subtraction, the whole numbers (integers) may be added separately and their sum or remainder joined to that of the fraction.

A complex fraction is one that contains another fraction in the numerator or in the divisor, or in both. It is simplified by following a process of division.

Thus:

$$\frac{2\frac{1}{4}}{3} = \frac{9}{4} \div 3 = \frac{\overset{3}{\cancel{9}}}{4} \times \frac{1}{\cancel{3}} = \frac{3}{4}$$

To change a decimal into a fraction, first note the number of decimal places, and then for the denominator write 1, followed by ciphers to equal the number of the decimals. The numerator of the fraction is the same as the decimal, but with the decimal point omitted.

$$.7 = \frac{7}{10}$$

$$.25 = \frac{25}{100} = \frac{1}{4}$$

$$.256 = \frac{256}{1000} = \frac{32}{125}$$

To change a common fraction to a decimal, it is necessary only to divide the numerator by the denominator.

$$\frac{25}{100} = 25 \div 100 = .25$$

NOTE

When a decimal fraction runs into many figures, it is often convenient to drop one or more of the figures. If a dropped figure is five or larger, one is added to the figure preceding it.

Thus, .837 would become .84, by dropping the 7, and adding 1 to the hundredths.

WEIGHTS AND MEASURES

CHAPTER VII

WEIGHTS AND MEASURES

THE common measure of weight is the avoirdupois, but the so-called troy weight is used for gold, silver, and precious stones.

In avoirdupois weight:

16 drams = 1 ounce
16 ounces = 1 pound
100 pounds = 1 hundredweight
20 hundredweight = 1 ton
112 pounds = 1 long hundredweight
2240 pounds = 1 long ton

In troy weight:

24 grains = 1 pennyweight
20 pennyweight = 1 ounce
12 ounces = 1 pound

In weighing precious stones, the unit is a carat, which is $3\frac{1}{5}$ grains:

In apothecaries' weight:

20 grains	= 1 scruple
3 scruples	= 1 dram
8 drams	= 1 ounce
12 ounces	= 1 pound (troy)

In linear measure, which is the measure of lines, the yard is the standard. The various units are as follows:

Linear Measures

12 inches	= 1 foot
3 feet	= 1 yard
$5\frac{1}{2}$ yards	= 1 rod
320 rods	= 1 mile

Any desired change from one unit of measure to another may be effected by either multiplication or division. For example, since there are twelve inches in one foot, and three feet in one yard, $12 \times 3 = 36$, which

is the number of inches in a yard. Conversely, since there are thirty-six inches in a yard, $36 \div 12 = 3$, which is the number of feet.

Since one rod contains five and one-half yards, and there are three hundred and twenty rods in a mile, $320 \times 5\frac{1}{2} = 1760$, which is the number of yards in a mile. Also, since a yard contains three feet, $1760 \times 3 = 5280$, which is the number of feet in a mile.

In going from the terms of a higher unit to a lower, multiplication is the means employed, while division serves when the change is from the lower to the higher.

When there are intermediate denominations, the process of multiplication or division must be repeated step by step through the various grades.

The square is the unit in measures of surface. A square inch is a square the sides of which are each one inch in length.

Surface Measures

144	square inches	=	1 square foot
9	square feet	=	1 square yard
$30\frac{1}{4}$	square yards	=	1 square rod
160	square rods	=	1 acre
640	acres	=	1 square mile

The manner of changing the terms in surface measure is the same as in linear measure, either by multiplication or division through the various steps. For example, if we wish to express in square rods the equivalent of 1840 square yards:

$$4840 \div 30\frac{1}{4} = 160$$

The 160 is the surface measure in square rods.

Volume Measures

Measures of volume are those having to do with length, breadth, and thickness. Here the unit is the cube. Where in the square inch we had the two measurements, of length and breadth, each an inch, we now have

three measurements, of length, breadth, and thickness, each an inch. And as for the inch, so for the other terms of measure.

Cubic Measures

1728 cubic inches	=	1 cubic foot
27 cubic feet	=	1 cubic yard
128 cubic feet	=	1 cord

It should be noted that the 128 cubic feet are not arranged so as to make the cord itself a cube. A cord of wood is 4 feet in width, 4 feet in height, and 8 feet in length.

The methods for reducing the terms of cubic measures are the same as those already explained. Thus, to reduce 243 cubic feet to cubic yards:

$$243 \div 27 = 9$$

Measures of Capacity

Measures of capacity are of two sorts, dry and liquid.

Dry Measures

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

Liquid Measures

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
$31\frac{1}{2}$ gallons = 1 barrel
2 barrels = 1 hogshead

Apothecaries' Liquid Measures

60 minims = 1 dram
8 drams = 1 ounce
16 ounces = 1 pint

Measures of Time

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
52 weeks = 1 year (less one day)
100 years = 1 century

A year contains 365 days, except a leap year, when one day is added, making the total 366.

The outer line of a circle is called the circumference. A straight line drawn from the circumference of a circle to its center is called the radius. A straight line passing from the circumference through the center to an opposite point on the circumference is the diameter. The radius equals one half of the diameter. The diameter equals two times the radius.

For the purposes of measurement, the circumference of the circle is divided into three hundred and sixty equal sections, with subdivisions of these into minutes and seconds.

Circular Measures

60 seconds = 1 minute
60 minutes = 1 degree
360 degrees = 1 circumference

The abbreviations for these are, for the seconds, ″; for the minutes, ′; for the degrees, °.

Angles are measured by means of the circumference of the circle. If in any circle a diameter be drawn vertically, and then another diameter be drawn horizontally, the circle will thus contain four right angles. These are measured by the number of degrees contained in the circumference between any two of these straight lines. Such sections of the circumference are called arcs. In this case, each arc measures one fourth of the total circumference of the circle. Since there are three hundred and sixty degrees in the circumference, this number divided by four gives ninety, which is the measure of the arc in each case, and is also the measure of the angle formed by the lines at the center. If another radius be drawn midway of any one of these four right angles, the right angle would be divided into two acute angles, each

measured by an arc of forty-five degrees. In like manner the arcs indicated by any two radii of the circle will give the measurement in degrees of the angle formed by such radii.

The diameter of a circle is contained in the circumference 3.1416 times. So, when the circumference is known, the diameter may be found by dividing it by 3.1416. Inversely, when the diameter is given, it multiplication by 3.1416 will give the circumference. The radius will of course serve in place of the diameter for this calculation, when doubled, since the radius is one half the diameter.

The area of a circle is found by multiplying the square of the radius by 3.1416.

In calculations not demanding exactness, $3\frac{1}{7}$ may be used as approximating 3.1416.

The surface of a globe is four times that of a circle having the same diameter. It can be determined by finding the product of 3.1416 and the square of the diameter.

Longitude and Time

Any imaginary straight line drawn around the earth so that it passes through the two poles is called a meridian.

Longitude is the name given to the distance between any two points on the earth's surface, eastward or westward. It is measured by the distance from a given meridian on the arc of the equator to another specified meridian. The standard from which longitude is reckoned in the United States and Great Britain is the meridian of Greenwich, in England. The difference in longitude between two places equals the arc of the equator between the meridians of the two. By reason of the revolution of the earth once in twenty-four hours, any place on the earth's surface describes in the twenty-four hours a circle of 360°. Owing to this fact, the longitude of any place may be expressed in terms of time as well as in terms of degrees, minutes, and seconds on the arc of the equator.

Any place on the earth's surface moves in one hour one twenty-fourth part of 360°, which is 15°. In a minute it moves one sixtieth part of 15°, which is 15′. In one second, it moves one sixtieth of 15′, which is 15″.

By analogical calculation, we find that any given spot on the earth's surface passes over one degree in 4 minutes, while it passes over 1′ in 4 seconds.

So, when we have longitude in degrees, if the number of degrees be multiplied by four, the product gives the corresponding number in minutes of time.

Conversely, when the longitude is expressed in minutes of time, if this number be divided by four, the quotient is the corresponding number of degrees.

On account of the revolution of the earth on its axis, the sun rises earlier at any point east of a given place, and later at all points to the westward. From this fact, it comes

that to determine the time of a place east from a place of which the time is known, we must add to the given time the difference in time between the two places; while in determining the time of a place to the westward, we must subtract from the given time the difference in time.

It being noon at Greenwich, we can calculate the longitude of a point where it is three o'clock in the morning.

Since at the latter place the hour is earlier, we know that it is west of Greenwich. We therefore subtract its time from that of Greenwich.

$$12 - 3 = 9$$

We then reduce this remainder in hours to minutes.

$$9 \times 60 = 540$$

To reduce the number of minutes to an equivalent in degrees, we must divide by four.

$$540 \div 4 = 135$$

Thus from the difference in time, three hours, we discover the distance in degrees of longitude.

Specific Gravity

An additional measure of weight developed by scientific research is specific gravity. This is determined in any substance by reckoning the number of times the weight of an equal volume of water is contained in the weight of the substance. A table of specific gravities will be found elsewhere in this volume.

The Metric System

In the metric system of weights and measures the standard is a meter. This is supposed to be the measure of one ten-millionth of the distance from the equator to the north pole. The length of the meter in inches is 39.37043.

The advantage of the metric system is that all the units are multiplied and divided

decimally, while the various prefixes to the word "meter" indicate the number of times the standard is multiplied or divided.

Linear Measure

1 millimeter	= .001 meter
1 centimeter	= .01 meter
1 decimeter	= .10 meter
1 dekameter	= 10 meters
1 hektometer	= 100 meters
1 kilometer	= 1000 meters
1 myriameter	= 10000 meters

In land measurements, a square dekameter is termed an ar. A square hektometer is termed a hektar.

In wood measure, a cubic meter is termed a ster.

In liquid measure, a cubic decimeter is termed a liter. Prefixes are used with this as with the word "meter," from milliliter (.001 liter) to kiloliter (1000 liters).

In weights, the unit is the gram, which equals the weight of a cubic centimeter of water. The prefixes are used also with the word, from a milligram (.001 gram) to a kilogram (1000 grams). 1000 kilograms make a metric ton.

Among the tables at the end of this volume will be found one giving equivalents in the metric and other measures.

RATIO AND PROPORTION

CHAPTER VIII

RATIO AND PROPORTION

THE relation as to size between two numbers is termed their ratio. For the purposes of such a comparison, the two numbers may be written as a fraction, but it is preferable to write them separated by a colon.

When such a relation between two numbers is compared with the relation between two other numbers, such comparison is called proportion, and it is indicated by a double colon.

$$3:6::5:10$$

This might be written:

$$\frac{3}{6} = \frac{5}{10}$$

That the proportion is correctly stated is witnessed by the fact that the fractional equation reduces to $\frac{1}{2} = \frac{1}{2}$.

The first and last numbers of a proportion are named the extremes, while the middle numbers are named the means.

When four numbers are thus in proportion, the fact may be verified by multiplying the extremes together and the means together. The products will be equal.

Thus, in the illustration given above:

$$3 : 6 :: 5 : 10$$
$$3 \times 10 = 6 \times 5$$
$$30 = 30$$

It is evident from this fact that either extreme is equal to the quotient obtained by dividing the product of the means by the other extreme. For example:

$$\frac{6 \times 5}{3} = 10$$

$$\frac{6 \times 5}{10} = 3$$

So, when three terms of a proportion are given, the fourth may be found in the manner

indicated above, which expresses what is called the Rule of Three.

$$5 : 15 :: 3 : ?$$

To discover the unknown extreme, we first multiply together the two means, and then divide the product by the given extreme.

$$15 \times 3 = 45$$

We divide this product by the known extreme.

$$45 \div 5 = 9$$

That this quotient is the correct answer may be proved by multiplying together the extremes, which will be found to equal the product of the means.

$$5 \times 9 = 15 \times 3$$
$$45 = 45$$

As an example of the simplicity and the usefulness of the Rule of Three, take the following:

At a certain hour of the day, a stake standing vertically nine feet above the ground

casts a shadow thirty feet in length horizontally. At the same hour, a stake fifteen feet vertically above the ground casts a shadow horizontally of how many feet?

$$9 : 15 :: 30 : ?$$

$$\frac{15 \times 30}{9} = 50$$

Thus 50 is the length of the second shadow.

In the statement of the proportion, the numbers should be so arranged as to give the unknown quantity the final place. The quantity corresponding with this should be in the ratio with it — that is, it should be the third term in the proportion. The other two corresponding quantities should form the first ratio of the proportion. Both ratios should follow the same order in reference to the size of the number. Thus, in the illustration above, the smaller number, nine, is followed by the larger number, fifteen. The other ratio follows the same order.

PERCENTAGE AND INTEREST

Chapter IX

PERCENTAGE AND INTEREST

PER CENT. is properly written with a period after cent., since it is an abbreviation for centum, meaning a hundred. Percentage is the term used to describe the calculation of the quantities when any unit is divided into a hundred parts. The fractional division of the dollar into cents illustrates the process very clearly. The dollar is divided into one hundred equal parts. One of these is therefore one per cent. of a dollar. Twenty-five cents is twenty-five per cent. of a dollar.

In this case, one dollar is the unit. If two dollars be taken as the unit, the figures in the percentage would be doubled. That is to say, one per cent. of two dollars is not one

cent, but is two cents. Twenty-five per cent. of two dollars is not twenty-five cents, but is fifty cents. In every case of percentage, the unit of the calculation is regarded as being divided into one hundred parts for working out any desired per cent.

The sign used for per cent. is %.

In a school where there had been one hundred pupils, there was an increase of twenty-five per cent. That made a total of one hundred and twenty-five pupils. In a school where there had been two hundred pupils, there was an increase of twenty-five per cent. The number of pupils was then two hundred and fifty. In a school where there were five hundred pupils, thirty per cent. had measles. Three hundred and fifty pupils, therefore, did not suffer from the disease.

Any per cent. of any number may be found by multiplying together the number and the per cent. expressed as a decimal.

Thirty per cent. of five hundred is found thus:

$$500 \times .30 = 150$$

In a town of 2680 population, there was an increase of 25% in a year. What was the new total of population?

$$2680 \times .25 = 670$$

$$2680 + 670 = 3350$$

Thus, 3350 was the population after the increase of 25%.

To find what per cent. any number is of a larger number, it is only necessary to divide the smaller by the larger. The quotient in decimals will be the per cent. required.

Thus, to determine what per cent. five dollars is of forty dollars:

$$5 \div 40 = .12\tfrac{1}{2}$$

The answer, then, is $12\frac{1}{2}$%.

Interest

Payment made for the use of borrowed money is called interest. The amount bor-

rowed is termed the principal. Interest is reckoned in terms of per cent. Interest at 6% means that six cents is to be paid annually for the use of each dollar borrowed. Similarly, 5% means five cents for each dollar; and any other per cent. analogously. One hundred dollars loaned for five years at six per cent. would amount to one hundred and thirty dollars.

In this instance:

$$.06 \times 5 = .30$$
$$100 \times .30 = 30$$
$$100 + 30 = 130$$

The product of the rate of interest and the term of the loan is multiplied by the amount of the loan. The result is then added to the original amount.

In calculating the amount of interest for fractional parts of a year, it is only necessary to take an equal fractional part of the interest rate. If the interest rate be 6%,

the interest on one dollar for one month will be $\frac{1}{12}$ of 6, which is $\frac{1}{2}$, or one half cent.

All of the problems having to do with interest are in reality questions concerning percentage, and they are to be solved in accordance with the instructions already given in this connection.

Compound interest is the term used when the interest, instead of being paid over, is regularly added to the principal, and thereafter itself draws interest as a part of the principal thus increased.

Here is a simple illustration of the method to be pursued in calculating compound interest:

The interest on a loan of three hundred dollars for a term of three years at six per cent. is to be compounded. The amount to be paid at the end of the term may be ascertained thus:

$$300 \times .06 = 18$$

This eighteen dollars, the interest for the first year, is now to be added to the principal of three hundred dollars, to form a new principal, on which the second year's interest is to be calculated.

$$300 + 18 = 318$$

$$318 \times .06 = 19.08$$

This nineteen dollars and eight cents is the interest on the new principal for the second year. It is now to be added to the principal, thus forming still another principal, on which the interest is to be calculated for the third year.

$$318 + 19.08 = 337.08$$

$$337.08 \times .06 = 20.22$$

This twenty dollars and twenty-two cents is the interest for the third year. By adding it to the last principal, we obtain the amount that is to be paid to the lender.

$$337.08 + 20.22 = 357.30$$

POWERS AND ROOTS

Chapter X

POWERS AND ROOTS

ANY number multiplied by itself gives the square of that number. The sign of such multiplication is a small figure two, written to the right of the number and above the line. Thus, 2^2 shows that two is to be multiplied by itself. This is called two square.

$$2^2 = 2 \times 2 = 4$$

Four is the square of two.

$$3^2 = 3 \times 3 = 9$$

Nine is the square of three.

Or, we may say, four is two raised to the second power; nine is three raised to the second power.

When two is multiplied by itself, and this product also is multiplied by two, the final product is the cube of two. The sign for

such multiplication is a small figure three. Thus:

$$2^3 = 2 \times 2 \times 2 = 8$$

Higher powers follow a like process.

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

This is also the square of four.

$$4^2 = 4 \times 4 = 16$$

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

This is also the square of nine:

$$9^2 = 9 \times 9 = 81$$

One who has learned the multiplication table knows already the square of each number up to twelve.

When any number is considered in reference to its square, it is termed the square root. For example, two is the square root of four; three is the square root of nine.

The other roots are analogous: Two is the cube root of eight; three is the cube root of twenty-seven.

The radical sign is used in connection with a power to indicate that its root is to be extracted. $\sqrt[2]{16}$ means that we are to find the square root of sixteen; $\sqrt[3]{27}$ means that we are to find the cube root of twenty-seven.

In extracting the square root, it should be borne in mind that, where a number contains one or two figures, the root will contain only one figure. Where a number contains three or four figures, the root will contain two figures; etc. For this reason, when we wish to find the square root of any number, we first separate it into periods of two figures each, beginning at the right. The units and tens thus form the first period; the hundreds and thousands, the second period, and so on. But the final period on the left may contain only one figure.

A small mark placed above the line between the figures is employed to show the separation of a number into periods.

To find the square root of 1369:

First, separate the number into periods, which will show that the root contains two figures.

13′69

Now, determine the highest square root of which the square is contained within thirteen, the period on the left. This must be three, of which the square is nine. Four would have sixteen for its square, which is not contained in thirteen. So, we set down three as the tens figure of the root.

$$13'69\,\underline{|3\,}$$

Next, we square this figure of the root — the tens figure. The square of three is nine. Nine, therefore, is to be subtracted from the thirteen. To the remainder, four, we add the other period, sixty-nine.

$$\begin{array}{l} 13'69\,\underline{|3\,} \\ \underline{\ 9\ } \\ \ 4\,69 \end{array}$$

We now double the tens figure of the root, which gives six. A cipher is added to the six to show that it is not a units figure. The number thus secured serves as a divisor for the remainder found above, four hundred and sixty-nine. The divisor is contained seven times in that remainder. We write seven as the second figure of the root. On substituting this seven for the cipher in the divisor, we have sixty-seven. This is contained in the dividend above, four hundred and sixty-nine, exactly seven times.

$$\begin{array}{r|l} 13'69 & \underline{37} \\ 9 & \\ 67 \;\; 4\,69 & \\ \underline{4\,69} & \end{array}$$

Thus it is found that the square root of 1369 is 37.

Or:

$$\sqrt[2]{1369} = 37$$

$$37^2 = 37 \times 37 = 1369$$

It will be found on examination that the square of any number containing two or more

figures is made up of the square of the tens + twice the tens × the units + the square of the units. In the example given above this formula is proven. The tens in the root squared make nine. Add to this the tens × the units, 3 × 7, which gives 21. This number doubled gives 42. The units figure, 7, squared gives 49. We now add these figures, 9 + 42 + 49. But in making this addition the order of the various numbers should be carefully observed.

Thus:

$$\begin{array}{r} 9 \\ 42 \\ 49 \\ \hline 1369 \end{array}$$

The addition shows that 1369 has 37 as its square root, for we have followed the formula: the square of a number equals the square of the tens + twice the tens × the units + the square of the units.

In extracting the square root of any number, this formula is practically reversed, and

it serves at every step to test the accuracy of the process.

Where there are more than two periods in a number, the method of extracting the square root is the same as that shown in the example above. The part of the root already found is regarded as so many tens, and is duly doubled, and used to serve as a divisor.

When the number is not a perfect square, the root may be extracted to any desired number of decimal places.

Where there are decimals in the square, the separation into periods begins at the decimal point and moves to the left as before for the integral number, but to the right for the decimal.

The number of periods to the right of the decimal point will show the number of decimal places in the root.

If a square has decimal places, the root will have one half as many.

The square root of a fraction is found by

extracting the square root of both numerator and denominator, and writing the result as a fraction.

Cube Root

The extraction of the cube root of any number is somewhat more difficult. The process depends on the fact that the cube of any number containing tens and units equals the sum of four parts. These four parts are the cube of the tens; three times the product of the square of the tens multiplied by the units; three times the product of the tens multiplied by the square of the units; the cube of the units.

The first step in extracting the cube root of a number is to separate it into periods, beginning at the right, but now each period must include three figures, in every case, except the final one on the left. The number of figures in the root will be the same as the number of the periods.

The formula bearing on the extraction of the cube root of any number is:

The cube of a number equals the tens cube + three times the product of the tens square × the units + three times the tens × the units square + the units cube.

It is convenient to abbreviate the word tens to t and the word units to u. Thus, the formula would run:

$$t^3 + 3(t^2 \times u) + 3(t \times u^2) + u^3.$$

By means of this formula, we may test the steps in extracting the cube root of any number.

Take the number 46,656. To extract the cube root, we first separate the number into periods.

46′656

We now examine the period 46, in order to learn the largest cube contained within it. This is readily seen to be 27. We therefore write 3, the cube root of 27, as the tens figure in the answer to our problem.

We next raise the number, 3, to the third power, which gives us 27 (hundreds), and subtract this from the 46, which gives a remainder of 19, to which the three figures of the other period in the cube are added. This remainder is 19656.

In the next step, 27, the cube of our first root figure, is used as a trial divisor for the remainder, 19656. We find that it will be contained 6 times, and we therefore write down 6 as the units figure of the root.

The accuracy of this is then proven by subtracting from 19656, $3 \times (t^2 \times u)$, which leaves a remainder of 3456.

$$3 \times (30^2 \times 6) = 16200$$
$$19656 - 16200 = 3456$$

The next step is to subtract from the above remainder $3 \times (u^2 \times t)$, which leaves a remainder in its turn of 324.

$$3 \times (u^2 \times t) = 3240$$
$$3456 - 3240 = 216$$

The final step is to subtract from this remainder u^3, which leaves no remainder.

$$u^3 = 216$$
$$216 - 216 = 0$$

By this absence of any remainder, we know that 36 is the cube root of 46,656.

The various stages in the calculation are shown in their order as follows:

$$\begin{array}{rr}
 & 46'656(36 \\
t^3 = & \underline{27} \\
 & 19656 \\
3 \times (t^2 \times u) = & \underline{16200} \\
 & 3456 \\
3 \times (u^2 \times t) = & \underline{3240} \\
 & 216 \\
u^3 = & 216
\end{array}$$

When the trial divisor is too large for the dividend, a cipher must be placed for the next figure in the root. A full period is then added to the dividend, and two ciphers to the divisor, after which the division proceeds.

Sometimes, the quotient will be found too large, in which case it must be reduced by one, and the necessary adjustments made.

When the number is not a perfect cube, the root may be extracted to as many decimal places as desired.

When a fraction has cubes for numerator and denominator, the cube root of each may be extracted and written as a fraction. When the fraction is not made up of cubes, it is best to reduce it to a decimal, and then extract the root to any number of decimal places desired.

LOGARITHMS

CHAPTER XI

LOGARITHMS

BY using exponents, any number may be represented as a power of ten. Such exponents are called logarithms.

$$
\begin{aligned}
10^0 &= 1\\
10^1 &= 10\\
10^2 &= 100\\
10^{-1} &= .1\\
10^{-2} &= .01
\end{aligned}
$$

From the above, it is shown that the logarithms corresponding are as follows:

log	1	=	0
"	10	=	1
"	100	=	2
"	.1	=	−1
"	.01	=	−2

(For convenience, the word "logarithm" is usually shortened to log.)

The logarithms of all numbers between 1 and 10 are 0 + a fraction; between 10 and 100, 1 + a fraction; between 100 and 1000, 2 + a fraction; between 1 and .1, −1 + a fraction.

The integral part of a logarithm is termed the characteristic; the fraction is termed the mantissa. This mantissa is always plus, even when the characteristic is minus. The minus sign is often written over the characteristic instead of before it.

From the fact that logarithms are really exponents, the sum of the logarithms of two numbers gives the logarithm of the product of those numbers.

$$\log 2 + \log 10 = \log 20$$

The logarithm of a power of a number equals the logarithm of the number multiplied by the exponents of the power.

The logarithm of a root of a number equals the logarithm of the number multiplied by the root's index.

The logarithm of a quotient equals the logarithm of the dividend less the logarithm of the divisor.

In the table of logarithms which follows, logarithms are given for all numbers under 1000. The characteristic and the decimal point are omitted, since the instructions above are sufficient as to these.

The logarithms of the digits from 1 to 9 are to be found in the tens column.

In using the table, the first two figures of the desired number are looked for in the left-hand column. The third figure is looked for in the top row. An imaginary line is then drawn from the figures in the left-hand column horizontally to the right; a line is drawn from the figure in the top row vertically downward. Where this line intersects the other line is the mantissa of the logarithm desired.

Table of Logarithms (*Concluded*)

N	0	1	2	3	4	5	6	7	8	9
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9943	9952
99	9956	9961	9965	9969	9978	9978	9983	9987	9991	9996

SPECIAL CONCISE RULES

Chapter XII

SPECIAL CONCISE RULES

THE area of a triangle = the base × the height.

In a right triangle, the side opposite the right angle is termed the hypothenuse.

The square of the hypothenuse = the sum of the squares of the other two sides.

The square of the hypothenuse − the square of one of the other sides equals the square of the third side.

The area of a sphere = the square of the diameter × 3.1416.

The volume of a sphere = $\frac{1}{6}$ the cube of the diameter × 3.1416.

The round surface of a cylinder = the circumference × the height.

The volume of a cylinder = the area of one end × the height.

The round surface of a cone = $\frac{1}{2}$ of the slant length × the circumference of the base.

The volume of a cone = $\frac{1}{3}$ the volume of a cylinder of equal base and vertical height.

The area of an ellipse = .7854 of its longest diameter × its shortest diameter.

The vibrations of a pendulum are inversely as the square root of the length.

Both the velocity and the quantity of water flowing through a hole are in proportion to the square root of the head.

The working force of the water is in proportion to the square root of the cube of the head.

Concerning the visibility of an object at sea, the square of the distance in miles equals $\frac{1}{4}$ the height of the object in feet.

Since sound travels about 1000 feet per second, the distance of a lightning flash will be given in feet by multiplying 1000 by the number of seconds after the flash before the sound of the thunder is heard.

Similarly in any case, where the cause of a sound is visible and the sound itself is audible.

TABLES

Linear Measures

12	inches	=	1 foot
3	feet	=	1 yard
$5\frac{1}{2}$	yards	=	1 rod
320	rods	=	1 mile

A line	= $\frac{1}{12}$ in.
A barleycorn	= $\frac{1}{3}$ in.
A hand (used in horse measure)	= 4 in.
A palm	= 3 in.
A span	= 9 in.
A cubit	= 18 in.
A military pace	= $2\frac{1}{2}$ ft.
A chain	= 4 rds.
A link	= $\frac{1}{100}$ chain
A furlong	= $\frac{1}{8}$ m.

A knot	= 6086 ft.
(used in navigation)	
A league	= 3 knots
A fathom	= 6 ft.
(used in measuring depths at sea)	
A cable-length	= 120 fathoms

Surface Measures

144	square inches	= 1 square foot
9	square feet	= 1 square yard
30¼	square yards	= 1 square rod
160	square rods	= 1 acre
10	square chains	= 1 acre
640	square acres	= 1 square mile
1	square mile	= 1 section
36	square miles	= 1 township

Volume Measures

1728	cubic inches	= 1 cubic foot
27	cubic feet	= 1 cubic yard
128	cubic feet	= 1 cord

40 feet of round timber	}	= 1 ton
50 feet of hewn timber	}	
40 cubic feet		= 1 ton of shipping

A cord is a pile of wood, 4 feet wide, 4 feet high, 8 feet long.

Capacity Measures

Dry

2 pints	=	1 quart
8 quarts	=	1 peck
4 pecks	=	1 bushel

Liquid

4 gills	=	1 pint
2 pints	=	1 quart
4 quarts	=	1 gallon
$31\frac{1}{2}$ gallons	=	1 barrel
2 barrels	=	1 hogshead

Weights

Troy

24	grains	= 1 pennyweight
20	pennyweights	= 1 ounce
12	ounces	= 1 pound
$3\frac{1}{2}$	grains	= 1 carat

Avoirdupois

16 drams	= 1 ounce
16 ounces	= 1 pound
100 pounds	= 1 hundred-weight
20 hundred-weight	= 1 ton
112 pounds	= 1 long hundred-weight
2240 pounds	= 1 long ton

Apothecaries' Weight

20 grains	= 1 scruple
3 scruples	= 1 dram
8 drams	= 1 ounce
12 ounces	= 1 pound

Apothecaries' Measure

60 minims	=	1 dram
8 drams	=	1 ounce
16 ounces	=	1 pint

Circular Measures

60 seconds	=	1 minute
60 minutes	=	1 degree
360 degrees	=	1 circumference

Names for Numbers

12 units	=	1 dozen
20 units	=	1 score
12 dozen	=	1 gross
12 gross	=	1 great gross

Paper Quantities

24 sheets	=	1 quire
20 quires	=	1 ream
2 reams	=	1 bundle
5 bundles	=	1 bale

Book Sheets

Folded in 2 leaves = 1 folio
Folded in 4 leaves = a quarto
Folded in 8 leaves = an octavo
Folded in 12 leaves = 1 duodecimo
Folded in 16 leaves = 1 16mo

Special Measures of Length

1 size = $\frac{1}{3}$ inch
1 hand = 4 inches
1 fathom = 6 feet
1 chain = 22 yards
(100 links)
1 furlong = 40 rods
1 knot = 6,086 feet
(nautical mile)

Household Liquid Measures

120 drops of water = 1 teaspoonful
60 drops thick liquid = 1 teaspoonful
2 teaspoonfuls = 1 desert-spoonful

3 teaspoonfuls	= 1 table-spoonful
16 table-spoonfuls	= 1 cup
1 cup	= ½ pint
1 pint	= 1 pound

Money

English

4	farthings	= 1 penny
12	pence	= 1 shilling
20	shillings	= { 1 pound, 1 sovereign
21	shillings	= 1 guinea
5	shillings	= 1 crown
2½	shillings	= ½ crown
2	shillings	= 1 florin

A sovereign is worth $4.86.

100 centimes (French)	= 1 franc ($0.19)
100 pfennings (German)	= 1 mark ($0.24)
100 kopecks (Russian)	= 1 rouble ($0.73)
100 kreutzers (Austrian)	= 1 florin ($0.45)

Equivalents

1	bushel of corn	= 56 pounds
1	bushel of rye	= 56 pounds
1	bushel of cornmeal	= 50 pounds
1	bushel of rye meal	= 50 pounds
1	bushel of wheat	= 60 pounds
1	bushel of potatoes	= 60 pounds
1	bushel of beans	= 60 pounds
1	bushel of oats	= 32 pounds
1	bushel of barley	= 48 pounds
1	stone of iron or lead	= 14 pounds
$21\frac{1}{2}$	stone of iron or lead	= 1 pig
8	pigs of iron or lead	= 1 fother
1	barrel of flour	= 196 pounds
1	barrel of pork	= 200 pounds
1	barrel of beef	= 200 pounds
1	barrel of lime	= 240 pounds
1	cental of grain	= 100 pounds
1	quintal of fish	= 100 pounds

Metric Equivalents

1 meter	= 39.37043 inches
1 kilometer	= 0.62138 mile
1 square meter	= 1550.031 square inches
1 hecktar	= 2.47110 acres
1 cubic centimeter	= 0.06103 cubic inch
1 cubic meter	= 1.30799 cubic yards
1 ster	= 0.27590 cord
1 liter	= 1.05671 liquid quarts
1 liter	= 0.90810 dry quart
1 milligram	= .015432 grain
1 gram	= 15.43235 grains
1 kilogram	= 2.20462 pounds
1 metric ton	= 2204.62 pounds

Temperature Scales

Fahrenheit freezing point of water	= 32 degrees
Fahrenheit boiling point of water	= 212 degrees
Centigrade freezing point of water	= zero

Centigrade boiling point of water	= 100 degrees
Reaumur freezing point of water	= zero
Reaumur boiling point of water	= 80 degrees

Specific Gravities

Acid, acetic	1.062
Acid, nitric	1.583
Acid, sulphuric	1.841
Air	.001292
Alcohol	.792
Aluminum	2.560
Brass	7.611
Brick	2.000
Coal, bituminous	1.250
Coal, anthracite	1.500
Copper	8.788
Cork	.240
Glass	2.760
Gold	19.258
Granite	2.720

Ice	.930
Iron, cast	7.150
Iron, wrought	7.770
Lead	11.350
Marble	2.720
Mercury	13.580
Milk	1.032
Platinum	21.841
Silver	10.500
Steel	7.816
Tin	7.291
Zinc	7.190

Roman Numerals

I	= 1
II	= 2
III	= 3
IV	= 4
V	= 5
VI	= 6
VII	= 7
VIII	= 8
IX	= 9

X	= 10
XI	= 11
XIX	= 19
XX	= 20
XXX	= 30
XL	= 40
L	= 50
LX	= 60
LXX	= 70
LXXX	= 80
XC	= 90
C	= 100
D (or IↃ)	= 500
M (or CIↃ)	= 1000
MM (or IIↃↃↃ)	= 2000
$\overline{\mathrm{V}}$ (or IↃↃ)	= 5000
$\overline{\mathrm{VI}}$ (or MMM)	= 6000
$\overline{\mathrm{X}}$ (or CCIↃↃ)	= 10000
$\overline{\mathrm{L}}$ (or IↃↃↃ)	= 50000
$\overline{\mathrm{LX}}$ (or MMMↃ)	= 60000
$\overline{\mathrm{C}}$ (or CCCIↃↃↃ)	= 100000
$\overline{\mathrm{M}}$ (or CCCCIↃↃↃↃ)	= 1000000

Compound Interest on $1.00 from One Year to Forty

Years	3%	3½%	4%	5%	6%
1	1.0300	1.0350	1.0400	1.0500	1.0600
2	1.0609	1.0712	1.0816	1.1025	1.1236
3	1.0927	1.1087	1.1249	1.1576	1.1910
4	1.1255	1.1475	1.1699	1.2155	1.2625
5	1.1593	1.1877	1.2167	1.2763	1.3382
6	1.1941	1.2293	1.2653	1.3401	1.4185
7	1.2299	1.2723	1.3159	1.4071	1.5036
8	1.2668	1.3168	1.3686	1.4775	1.5938
9	1.3048	1.3629	1.4233	1.5513	1.6895
10	1.3439	1.4106	1.4802	1.6289	1.7908
11	1.3842	1.4600	1.5395	1.7103	1.8983
12	1.4258	1.5111	1.6010	1.7959	2.0122
13	1.4685	1.5640	1.6651	1.8856	2.1329
14	1.5126	1.6187	1.7317	1.9799	2.2609
15	1.5580	1.6753	1.8009	2.0789	2.3966
16	1.6047	1.7340	1.8730	2.1829	2.5404
17	1.6528	1.7947	1.9479	2.2920	2.6928
18	1.7024	1.8575	2.0258	2.4066	2.8543
19	1.7535	1.9225	2.1068	2.5270	3.0256
20	1.8061	1.9898	2.1911	2.6533	3.2071

COMPOUND INTEREST ON $1.00 FROM ONE YEAR TO FORTY (*Continued*)

Years	3%	3½%	4%	5%	6%
21	1.8603	2.0594	2.2788	2.7860	3.3996
22	1.9161	2.1315	2.3699	2.9253	3.6035
23	1.9736	2.2061	2.4647	3.0715	3.8197
24	2.0328	2.2833	2.5633	3.2251	4.0489
25	2.0938	2.3632	2.6658	3.3864	4.2919
26	2.1566	2.4460	2.7725	3.5557	4.5494
27	2.2213	2.5316	2.8834	3.7335	4.8223
28	2.2879	2.6202	2.9987	3.9201	5.1117
29	2.3566	2.7119	3.1187	4.1161	5.4184
30	2.4273	2.8068	3.2434	4.3219	5.7435
31	2.5001	2.9050	3.3731	4.5380	6.0881
32	2.5751	3.0067	3.5081	4.7649	6.4534
33	2.6523	3.1119	3.6484	5.0032	6.8406
34	2.7319	3.2209	3.7943	5.2533	7.2510
35	2.8139	3.3336	3.9461	5.5160	7.6861
36	2.8983	3.4503	4.1039	5.7918	8.1473
37	2.9852	3.5710	4.2681	6.0814	8.6361
38	3.0748	3.6960	4.4388	6.3855	9.1543
39	3.1670	3.8254	4.6164	6.7048	9.7035
40	3.2620	3.9593	4.8010	7.0400	10.2857

AMOUNT FROM $1.00 DEPOSITED ANNUALLY

Years	3%	3½%	4%	5%	6%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0300	2.0350	2.0400	2.0500	2.0600
3	3.0909	3.1062	3.1216	3.1525	3.1836
4	4.1836	4.2149	4.2465	4.3101	4.3746
5	5.3091	5.3625	5.4163	5.5256	5.6371
6	6.4684	6.5502	6.6330	6.8019	6.9753
7	7.6625	7.7794	7.8983	8.1420	8.3938
8	8.8923	9.0517	9.2142	9.5491	9.8975
9	10.1591	10.3685	10.5828	11.0266	11.4913
10	11.4639	11.7314	12.0061	12.5779	13.1808
11	12.8078	13.1420	13.4864	14.2068	14.9716
12	14.1920	14.6020	15.0258	15.9171	16.8699
13	15.6178	16.1130	16.6268	17.7130	18.8821
14	17.0863	17.6770	18.2919	19.5986	21.0151
15	18.5989	19.2957	20.0236	21.5786	23.2760
16	20.1569	20.9710	21.8245	23.6575	25.6725
17	21.7616	22.7050	23.6975	25.8404	28.2129
18	23.4144	24.4997	25.6454	28.1324	30.9057
19	25.1169	26.3572	27.6712	30.5390	33.7600
20	26.8704	28.2797	29.7781	33.0660	36.7856

AMOUNT FROM $1.00 DEPOSITED ANNUALLY
(Continued)

Years	3%	3½%	4%	5%	6%
21	28.6765	30.2695	31.9692	35.7193	39.9927
22	30.5368	32.3289	34.2480	38.5052	43.3923
23	32.4529	34.4604	36.6179	41.4305	46.9958
24	34.4265	36.6665	39.0826	44.5020	50.8156
25	36.4593	38.9499	41.6459	47.7271	54.8645
26	38.5530	41.3131	44.3117	51.1135	59.1564
27	40.7096	43.7591	47.0842	54.6691	63.7058
28	42.9309	46.2906	49.9676	58.4026	68.5281
29	45.2189	48.9108	52.9663	62.3227	73.6398
30	47.5754	51.6227	56.0849	66.4388	79.0582
31	50.0027	54.4295	59.3283	70.7608	84.8017
32	52.5028	57.3345	62.7015	75.2988	90.8898
33	55.0778	60.3412	66.2095	80.0638	97.3432
34	57.7302	63.4532	69.8579	85.0670	104.1838
35	60.4621	66.6740	73.6522	90.3203	111.4348
36	63.2759	70.0076	77.5983	95.8363	119.1209
37	66.1742	73.4579	81.7022	101.6281	127.2681
38	69.1594	77.0289	85.9703	107.7095	135.9042
39	72.2342	80.7249	90.4091	114.0950	145.0585
40	75.4013	84.5503	95.0255	120.7998	154.7620

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